On brane-antibrane forces

Jian-Xin Lu, Bo Ning and Shan-Shan Xu

Interdisciplinary Center for Theoretical Study,
University of Science and Technology of China,
Hefei, Anhui 230026, China
E-mail: `jxlu@ustc.edu.cn`, `nwaves@mail.ustc.edu.cn`, `xuss@mail.ustc.edu.cn`

Shibaji Roy

Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Calcutta-700 064, India
E-mail: `shibaji.roy@saha.ac.in`

ABSTRACT: In this note, we will discuss two aspects of brane-antibrane forces. In one aspect, we generalize the force calculation of D0-$\bar{\rm D}$0 of Banks and Susskind to Dp-$\bar{\rm D}p$ for $1 \leq p \leq 8$. In particular, we find that the force is also divergent for $p = 1$ while for the other cases ($p \geq 2$) the forces are in general finite when $Z \to 0^+$, where $Z = \frac{Y^2}{2\pi^2\alpha'} - 1$ with Y, the brane-antibrane separation. However, the forces are divergent for all cases when $Z < 0$, signalling the occurrence of open string tachyon condensation in this regime. The other deals with the puzzling static nature of the supergravity brane-antibrane configurations. We will show that the force on a brane probe due to a brane-antibrane background vanishes when the probe is placed at the location of the coincident brane-antibranes, thereby providing a direct evidence for the existence of general static brane-antibrane configuration in the supergravity approximation.

Contents

1. Introduction

The brane-antibrane systems in type II superstrings break all spacetime supersymmetries. Consequently, unlike the so-called BPS branes, their dynamics, even though more interesting, is difficult to understand given our limited knowledge of the underlying full theory. A coincident D-brane-antiD-brane pair (or a non-BPS D-brane) in type II theories is unstable due to the presence of tachyonic mode in the weakly coupled open string description [1], however, it should be noted that the corresponding strongly coupled system can be very complicated and the underlying dynamics may be different [2–4]. As a result, these systems decay and the decay occurs by the process known as tachyon condensation [5]. The tachyon condensation is well understood by now in the open string description using either the string field theory approach [6–8] or the tachyon effective action approach [9] on the brane. The closed string (or supergravity) approach on this process and the related issues have also been discussed in [10–13]. In this approach, we interpret the known non-supersymmetric ten-dimensional type II supergravity solutions [12, 14–18] as coincident brane-antibrane systems and relate the parameters in the solutions to the corresponding microscopic parameters such as the number of branes, the number of antibranes and the tachyon parameter. Using these relations, we have calculated the ADM mass and have shown that the solution and the ADM mass capture all the required properties and give a correct description of the tachyon condensation [11, 12] as advocated by Sen [5] on the D-$\bar{\text{D}}$ system.

In this note, we will discuss two issues on the brane-antibrane systems, one in the open string description and the other in the supergravity (or closed string) description. One possible way to get a signal of the occurrence of the tachyon condensation is through calculating the brane-antibrane force at a given separation and examine the force behavior as the separation approaches the string scale as Banks and Susskind did in [19] for D0-$\bar{\text{D}}$0 system. We will generalize Banks and Susskind's analysis of $p = 0$ to $1 \leq p \leq 8$[1] in the following section. We will see that $p = 1$ case is similar to $p = 0$, i.e., the force between

---

[1]To make sense of a separation between two Dp branes, we need to limit $p \leq 8$ since $p = 9$ is a spacetime filling brane.

the brane and the antibrane is divergent, while for other $p \geq 2$ the story is different and the force is actually finite, when $Z \to 0^+$ where $Z = \frac{Y^2}{2\pi^2\alpha'} - 1$ with Y the brane-antibrane separation. However, when $Z < 0$, the force is always divergent (for all p), indicating the occurrence of the tachyon condensation in that regime, since the divergence is actually due to the tachyon mode of the open string connecting the Dp and the $\bar{\text{D}}p$. The divergence of the Born-approximated force between the brane and the antibrane at a separation of string scale indicates such a description breaks down but at the same time, the appearance of such a violent force may also indicate the occurrence of a new process and we know that this corresponds to the tachyon condensation. So we may use the appearance of such violent force as an evidence for the occurrence of the tachyon condensation.

The interpretation of non-supersymmetric static solutions of ten dimensional type II supergravities as representing the coincident brane-antibrane systems has a puzzle. One in general expects the non-existence of such a static configuration since the system under consideration is unstable. Therefore the static nature of these solutions must be due to the supergravity approximation. This static nature has an advantage in that it enables us to use the ADM mass (calculated asymptotically) to capture the off-shell tachyon potential as mentioned in [10] and discussed further in [20] even though the small distance behavior of such solutions may not be trusted in general. In other words, the off-shell tachyon potential can be represented by a continuous family of ADM mass which corresponds to a family of static supergravity configurations labeled by the mass parameter. In this sense, we can discuss the tachyon condensation semi-empirically if the parameters in the solutions can be related to the number of branes, the number of antibranes and the tachyon parameter as was achieved successfully in [11, 12]. How to understand the static nature of the non-supersymmetric supergravity solutions has been discussed in [10]. It was argued there that these solutions can be static even if the brane sources are time dependent, in analogy with the static exterior geometry of a pulsating spherically symmetric star, thanks to Birkhoff's theorem, and the time-dependence could presumably be discerned to the level of higher mass modes of closed string. This has been further addressed in [20] for the case of chargeless configurations by considering the relation between the disappearance of conical singularity and the vanishing force between the coincident brane-antibrane in the supergravity approximation. In section 3, we will use a probe approach to show that when a probe reaches the location of the coincident branes-antibranes but still not strongly bounded to the brane-antibrane system (in other words, the probe can still be taken as a probe), the force between the probe and the system vanishes, therefore providing a more direct evidence for the static nature of a general brane-antibrane system in the supergravity approximation.

2. The analysis of brane-antibrane force

We consider weakly coupled type II strings in ten dimensions. These theories admit various BPS Dp branes with p even in IIA theories and odd in IIB theories. The brane tension is inversely proportional to the string coupling g and as such in the weak-coupling limit, i.e. $g \to 0$, the tension will be divergent. So, one may naively expect that the brane may no

longer be taken as rigid and flat and neither the spacetime can be flat any more in contrary to what we usually do in the perturbative calculations. In the following, we will show that the naive expectation is wrong even up to the distance of the order of string scale. For this, let us first consider the metric of the Dp brane supergravity configurations as [23, 24],

$$ds^2 = \left(1 + \frac{k_p}{r^{7-p}}\right)^{-\frac{7-p}{8}} dx_\parallel^2 + \left(1 + \frac{k_p}{r^{7-p}}\right)^{\frac{p+1}{8}} dx_\perp^2, \qquad (2.1)$$

where r is the radial distance transverse to the brane, $x_\parallel$ are the directions along the branes and $x_\perp$ are those transverse to the branes. For asymptotically-flat and well-behaved supergravity configurations, we need to take $p = 0, 1, \cdots 6$. The parameter k_p is related to the ten dimensional Newton constant $2\kappa^2$, the number of Dp branes N and the Dp-brane tension T_p, apart from some numerical factor (which are irrelevant to the following discussion and will be ignored), as

$$k_p \sim 2\kappa^2 N T_p \sim N g \alpha'^{(7-p)/2}, \qquad (2.2)$$

where we have expressed $2\kappa^2$ and T_p in terms of α' and g as given, for example, in [21]. From this, it is clear that for a large but fixed N and fixed α', k_p vanishes as $g \to 0$ and therefore the spacetime remains flat even for $\alpha'^{1/2} \gg r \gg (Ng)^{1/(7-p)}\alpha'^{1/2}$. So if we don't probe a distance much smaller than the string scale, we are safe to take both the brane and the spacetime as flat in the lowest order calculation.

The calculation of the interaction (amplitude) between two parallel Dp branes separated by a distance Y can be computed (for example as given in [22]) in the lowest order as an open string one-loop annulus diagram with one end of the open string located at one Dp brane and the other end at the other Dp brane. This can also be viewed as a tree-level closed string amplitude, creating a closed string at one Dp brane, propagating a distance Y and then being absorbed by the other D-brane at the other end. The interaction amplitude has two contributions, one from the NSNS closed string exchange and the other from the RR closed string exchange. It is

$$A = V_{p+1} \int_0^\infty \frac{dt}{t} (2\pi t)^{-\frac{(p+1)}{2}} e^{-\frac{tY^2}{8\pi^2\alpha'^2}} \prod_{n=1}^\infty \left(1 - q^{2n}\right)^{-8}$$
$$\frac{1}{2}\left\{-16 \prod_{n=1}^\infty \left(1 + q^{2n}\right)^8 + q^{-1}\prod_{n=1}^\infty \left(1 + q^{2n-1}\right)^8 - q^{-1}\prod_{n=1}^\infty \left(1 - q^{2n-1}\right)^8\right\}, \quad (2.3)$$

where V_{p+1} is the p-brane worldvolume, $q = e^{-t/4\alpha'}$ and the integration variable t is the proper time in the open string channel. In the above, the first two terms in the curly bracket are from the NSNS closed string sector exchange while the the third term is from the RR sector. The BPS nature of this interaction tells that the amplitude actually vanishes which can also be seen from the above two NSNS terms canceling the third RR term using the "usual abstruse identity". The interaction for a Dp brane and an anti Dp-brane placed parallel at a separation Y can be obtained from the above simply by switching the sign in front of the RR term and the amplitude is therefore just twice the absolute value of the

RR term and is given as

$$\mathcal{A} \equiv \frac{A}{V_{p+1}} = \int_0^\infty \frac{dt}{t} \left(2\pi t\right)^{-\frac{(p+1)}{2}} e^{-\frac{t}{4\alpha'}\left(\frac{Y^2}{2\pi^2\alpha'}-1\right)} \prod_{n=1}^\infty \left(\frac{1-q^{2n-1}}{1-q^{2n}}\right)^8, \tag{2.4}$$

where we have defined $\mathcal{A}$, the interaction amplitude per unit p-brane volume and as in [19], we introduce the parameter Z as

$$Z = \frac{Y^2}{2\pi^2\alpha'} - 1 \tag{2.5}$$

and the function

$$g(t) = \prod_{n=1}^\infty \left(\frac{1-q^{2n-1}}{1-q^{2n}}\right)^8. \tag{2.6}$$

One can show using the relations for θ-functions that $g(t) \to 1$ as $t \to \infty$ while $g(t) \to (t/2\pi\alpha')^4$ as $t \to 0$. These limits will be needed later. For simplicity, let us define a variable $u = t/4\alpha'$ and the attractive force per unit p-brane volume is now

$$f = -\frac{d\mathcal{A}}{dY} = \frac{Y}{\pi^2\alpha'(8\pi\alpha')^{(p+1)/2}} \int_0^\infty du\, u^{-\frac{p+1}{2}} e^{-uZ} g(u), \tag{2.7}$$

where

$$g(u) = \begin{cases} 1 & u \to \infty \\ (2u/\pi)^4 & u \to 0 \end{cases} \tag{2.8}$$

and $0 < g(u) < 1$ in general. For $Z > 0$, the only possible divergence for the above force comes from $u \to 0$ and one can show using the limiting expression for $g(u)$ in (2.8) for $u \to 0$ that the integration is actually convergent there for those allowed $0 \le p \le 8$. Therefore the attractive force is finite as expected since no new process such as tachyon condensation occurs when the brane-antibrane separation is larger than the string scale.

We now examine the force behavior when $Z \to 0^+$. For this, let us change the integration variable to $v = Zu$, we have now

$$\begin{aligned} f &\sim Z^{\frac{p-1}{2}} \int_0^\infty dv\, v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \\ &= Z^{\frac{p-1}{2}} \left[\int_0^{aZ} dv\, v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) + \int_{aZ}^\infty dv\, v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right)\right] \\ &\ge Z^{\frac{p-1}{2}} \int_{aZ}^\infty dv\, v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \\ &\approx Z^{\frac{p-1}{2}} \int_{aZ}^\infty dv\, v^{-\frac{p+1}{2}} e^{-v}, \end{aligned} \tag{2.9}$$

where a is a fixed large number ($\gg 1$) and in the last line we have used $g(u) \to 1$ for large u. Let us examine the integration in the last line above when $Z \to 0^+$. For $p = 0$, the integration is $\Gamma(1/2)$, finite, and the force $f \ge 1/\sqrt{Z} \to \infty$ as discussed by Banks and Susskind in [19]. For $p = 1$, the force $f \ge \Gamma(0) \to \infty$ is also divergent. Therefore this case is similar to the $p = 0$ case. For $p \ge 2$, the above expression for the integration appears as $0 \cdot \infty$ and we need a more careful analysis than the above.

For this, let us re-express the force as

$$f \sim Z^{\frac{p-1}{2}} \int_0^\infty dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \tag{2.10}$$

$$= Z^{\frac{p-1}{2}} \left[\int_0^{bZ} dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) + \int_{bZ}^{aZ} dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) + \int_{aZ}^\infty dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \right]$$

$$= Z^{\frac{p-1}{2}} \left[\int_0^{bZ} dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) + \int_{aZ}^\infty dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \right] + \int_b^a du u^{-\frac{p+1}{2}} e^{-uZ} g(u).$$

where we have introduced two fixed parameters b and a with $b \ll 1$ while $a \gg 1$. The last term in the last line of eq. (2.10) corresponds to the second term of the second line of the same equation. However, note that we have expressed it in terms of the original integration variable u. Since both b and a are fixed, so the last term in the last line in (2.10) should be finite. Let us examine the first term in the square bracket with the pre-factor $Z^{(p-1)/2}$ in the last line in (2.10). Since b is very small, so we can approximate the function $g(v/Z) \sim (v/Z)^4$ (as given in eq. (2.8)) in the integration. With this, one can show

$$Z^{\frac{p-1}{2}} \int_0^{bZ} dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \sim Z^{\frac{p-1}{2}} \int_0^{bZ} dv v^{-\frac{p+1}{2}} e^{-v} (v/Z)^4$$
$$\sim b^{(9-p)/2}, \tag{2.11}$$

i.e., finite. Let us examine the second term with the pre-factor now. For very large a, we have

$$Z^{(p-1)/2} \int_{aZ}^\infty dv v^{-\frac{p+1}{2}} e^{-v} g\left(\frac{v}{Z}\right) \sim Z^{(p-1)/2} \int_{aZ}^\infty dv v^{-\frac{p+1}{2}} e^{-v}$$
$$= Z^{(p-1)/2} \left[\int_1^\infty dv v^{-\frac{p+1}{2}} e^{-v} + \int_{aZ}^1 dv v^{-\frac{p+1}{2}} e^{-v} \right]$$
$$< a^{(1-p)/2}, \tag{2.12}$$

therefore also finite as $Z \to 0^+$. Here we have used $g(u) \to 1$ for large u in the first line above. Also it is obvious that the first term in the square bracket in the second line above is finite and the second term is

$$\int_{aZ}^1 dv v^{-\frac{p+1}{2}} e^{-v} < \int_{aZ}^1 dv v^{-\frac{p+1}{2}} \sim (aZ)^{(1-p)/2}. \tag{2.13}$$

So the force between the brane and the antibrane is finite when $Z \to 0^+$ for $2 \le p \le 8$.

In summary, we have seen that the force between the brane and the antibrane is divergent for $p = 0, 1$ while it is finite for $2 \le p \le 8$ when $Z \to 0^+$. Further, the force is always divergent for $0 \le p \le 8$ when $Z < 0$. The above divergences are due to large u contribution to the force given in (2.7) and can be understood by writing the large u behavior as $\int^\infty du u^{-\frac{p+1}{2}} e^{-Zu}[1 + \mathcal{O}(e^{-u})]$. Now it is clear that the large u integration diverges when $p = 0, 1$ while it converges for $2 \le p \le 8$ when $Z = 0$. When $Z < 0$, the exponential e^{-Zu} in the integration dominates and dictates the large u divergence for all $0 \le p \le 8$. These divergences are due to the tachyon mode of the open string connecting

the Dp and the $\bar{\text{D}}p$ as can be seen from the expansion of $g(u) = 1 + \mathcal{O}(e^{-u})$ for large u where the first term '1' corresponds to the tachyon mode contribution. As discussed in the Introduction, the appearance of such a violent force indicates the breakdown of the calculation or it can be thought of as an indication of a new process, therefore, signalling the occurrence of the tachyon condensation.

Another way to understand the connection between the force divergence and the onset of tachyonic instability is as follows: The force divergence implies the appearance of certain pole in the force calculation. But this divergence occurs either at space-like separation $Y^2 \leq 2\pi^2\alpha'$ for $p = 0, 1$ or at space-like separation $Y^2 < 2\pi^2\alpha'$ for all p which implies that the corresponding pole is a tachyon i.e. we see the onset of tachyonic instability, since only a tachyonic pole can propagate in a space-like separation.

The above discussion implies that the initiation of tachyonic instability for brane-antibrane systems is different for $p \leq 1$ and for $p > 1$. For $p \leq 1$, this occurs at a larger brane separation and the onset of tachyonic instability at the beginning is milder (only a power divergence). For $p > 1$, the instability begins at $Z < 0$ and it is much stronger (an exponential divergence). However, in this region the nature of the tachyonic instability is essentially the same for both $p \leq 1$ and $p > 1$ cases. In other words, the brane-antibrane system starting annihilation or tachyon condensation takes place at a larger brane separation for $p \leq 1$ case than for $p > 1$ case. Whether there is a deep reason or implication behind this difference remains to be seen.[2]

3. Evidence for the static nature of non-susy solutions

The static, non-supersymmetric and asymptotically flat p-brane solutions[3] having isometries $\text{ISO}(p, 1) \times \text{SO}(d - p - 1)$ of type II supergravities in arbitrary space-time dimensions (d) are given in [14, 15, 17]. For the purpose of this paper, we take $d = 10$ in the following discussion. Unlike the BPS p-branes characterized by a single unknown parameter, these solutions are characterized by three unknown parameters and could be either charged or chargeless with respect to a $(p+1)$-form gauge field. These non-susy p-branes have a natural interpretation as coincident p-brane-anti-p-brane (or non-BPS p-brane) [10–12, 20]. As mentioned in the Introduction, this interpretation has a puzzle since one would expect the non-existence of such static solutions given the unstable nature of these systems. We will use a brane probe approach in this section to understand such static nature of these configurations in the supergravity approximation.

The static non-supersymmetric p-brane solutions representing coincident p-brane-anti

---

[2]We thank the referee for emphasizing to us the curious dependence on p for the onset of tachyonic instability in the $Z \to 0^+$ limit which led to this discussion.

[3]We use the terminology non-susy p-brane to represent generically either the p-brane-anti$p$-brane system or the non-BPS p-branes.

p-brane systems in ten dimensional type II supergravities are [17]

$$ds^2 = F^{-\frac{7-p}{8}}\left(-dt^2 + dx_1^2 + \ldots + dx_p^2\right) + F^{\frac{p+1}{8}}\left(H\tilde{H}\right)^{\frac{2}{7-p}}\left(dr^2 + r^2 d\Omega_{8-p}^2\right)$$

$$e^{2\phi} = F^{-a}\left(\frac{H}{\tilde{H}}\right)^{2\delta}$$

$$A_{[p+1]} = -\sinh\theta\cosh\theta\left(\frac{C}{F}\right)dx^0 \wedge \ldots \wedge dx^p \tag{3.1}$$

where we have expressed the metric in Einstein frame. In the above,

$$F = \cosh^2\theta\left(\frac{H}{\tilde{H}}\right)^{\alpha} - \sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^{\beta}$$

$$C = \left(\frac{H}{\tilde{H}}\right)^{\alpha} - \left(\frac{\tilde{H}}{H}\right)^{\beta}$$

$$H = 1 + \frac{\omega^{7-p}}{r^{7-p}}, \qquad \tilde{H} = 1 - \frac{\omega^{7-p}}{r^{7-p}} \tag{3.2}$$

with the parameter relation

$$b = (\alpha + \beta)(7 - p)\, g\, \omega^{7-p} \sinh 2\theta \tag{3.3}$$

Here α, β, θ, and ω are integration constants, g is the string coupling and $a = (p-3)/2$. Also α and β can be solved, for the consistency of the equations of motion, in terms of δ as

$$\alpha = \sqrt{\frac{2(8-p)}{7-p} - \frac{(7-p)(p+1)}{16}\delta^2} + \frac{a\delta}{2}$$

$$\beta = \sqrt{\frac{2(8-p)}{7-p} - \frac{(7-p)(p+1)}{16}\delta^2} - \frac{a\delta}{2}. \tag{3.4}$$

These two equations indicate that the parameter δ is bounded as

$$|\delta| \le \frac{4}{7-p}\sqrt{\frac{2(8-p)}{p+1}}. \tag{3.5}$$

The solution (3.1) is therefore characterized by three parameters δ, ω and θ.

As demonstrated successfully in [11], once the three parameters of the above solutions are expressed in terms of the number of Dp-branes (N), the number of anti Dp branes ($\bar{N}$) and tachyon parameter T, the tachyon condensation process can be described correctly. In particular, we have the parameter δ

$$\delta = \frac{a}{|a|}\sqrt{\frac{8-p}{2(7-p)}}\left[|a|\sqrt{\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}} - \sqrt{a^2\left(\cos^2 T + \frac{(N-\bar{N})^2}{4N\bar{N}\cos^2 T}\right) + 4\sin^2 T}\right], \tag{3.6}$$

– 7 –

and the ADM mass

$$M(N, \bar{N}, T) = T_p \sqrt{(N + \bar{N})^2 - 4N\bar{N}(1 - \cos^4 T)}, \qquad (3.7)$$

with $0 \leq T \leq \pi/2$ and T_p, the single Dp brane tension. We have in general $M = (N + \bar{N})T_p + V(T)$ with $V(T)$ the tachyon potential. Just at the start of tachyon condensation, i.e., $T = 0$, we should have $V(T = 0) = 0$ and $M = (N + \bar{N})T_p$. The expression in (3.7) for M indeed satisfies this. As discussed in [10], the parameter δ vanishes at $T = 0$ and the above expression in (3.6) satisfies this, too. In order to examine whether the force between the Dp and the $\bar{\text{D}}p$ in the coincident Dp-$\bar{\text{D}}p$ system vanishes or not using a probe Dp or a probe $\bar{\text{D}}p$, we need to place the probe at the location of the coincident Dp-$\bar{\text{D}}p$ branes, i.e., at $r = \omega$ as implied in (3.1). While at the same time, we need to make sure that the probe remains as a probe, i.e., not strongly bounded to the original coincident Dp-$\bar{\text{D}}p$ system. This can be so only at the start of the tachyon condensation since there $M(N, \bar{N}, T) = (N + \bar{N})T_p$, or at least close to the top of the tachyon potential where $M(N + 1, \bar{N}, T) \approx M(N, 1 + \bar{N}, T) \approx T_p + M(N, \bar{N}, T)$. In other words, the value of the parameter δ should be very close to its initial vanishing value at the start of tachyon condensation. This will be the key point for us to show in the following that the probe does indeed feel 'no force' when placed at the $r = \omega$.

For definiteness, let us consider a Dp-brane probe placed at a radial distance $r \geq \omega$ and parallel to the brane directions $x_\parallel$ in the static non-susy Dp brane background (3.1). Our purpose is to calculate the interaction potential and for this we just need to consider the bosonic worldvolume action for the probe and freeze the worldvolume excitations as usually done for a p-brane probe placed in a BPS p-brane background in [24]. The bosonic Lagrangian density for a Dp probe placed along $x_\parallel$ without worldvolume excitations is

$$\mathcal{L}_p = -T_p \left[e^{-\phi} \sqrt{-\det \gamma_{\mu\nu}} + A_{01\cdots p} \right] \qquad (3.8)$$

where[4] $\gamma_{\mu\nu} = g_{\mu\nu}$ with $g_{\mu\nu}$ the background metric (3.1) along $x_\parallel$ directions but now in the string frame, and $A_{01\cdots p}$ is the corresponding Ramond-Ramond background potential. Here we have set the worldvolume coordinates $\sigma^\mu = X^\mu$ with $\mu = 0, 1, \cdots p$ and frozen the worldvolume excitations. From the relation between string frame metric and the one given in (3.1) which is in Einstein frame, we have now

$$g_{\mu\nu} = e^{\frac{\phi}{2}} F^{-\frac{7-p}{8}} \eta_{\mu\nu} = F^{-\frac{a}{4} - \frac{7-p}{8}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta}{2}} \eta_{\mu\nu} = F^{-\frac{1}{2}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta}{2}} \eta_{\mu\nu}, \qquad (3.9)$$

where we have used the explicit background in (3.1) and $a = (p-3)/2$. The potential

---

[4]If the probe is anti Dp brane, we just need to change the sign in front of $A_{01\cdots p}$ and the conclusion will remain the same.

density (or the potential per p-brane volume) can be calculated using background (3.1) as

$$V_p = T_p \left[e^{-\phi} \sqrt{-\det \gamma_{\mu\nu}} + A_{01\cdots p} \right]$$

$$= T_p \left[F^{-1} \left(\frac{H}{\tilde{H}} \right)^{\frac{a\delta}{2}} - \sinh\theta\cosh\theta \left(\frac{C}{F} \right) \right]$$

$$= T_p \frac{\left(\frac{\tilde{H}}{H} \right)^{\alpha - \frac{a\delta}{2}} - \sinh\theta\cosh\theta(1 - \left(\frac{\tilde{H}}{H} \right)^{\alpha+\beta})}{\cosh^2\theta - \left(\frac{\tilde{H}}{H} \right)^{\alpha+\beta} \sinh^2\theta}, \tag{3.10}$$

where we have used the expressions for F and C in (3.2) in the last line above. Note that from (3.4), $\alpha - a\delta/2 \geq 0$ and $\alpha + \beta \geq 0$,[5], therefore the potential density remains finite for $r \geq \omega$ as can be seen from the explicit expressions of H and $\tilde{H}$ and their dependences on the radial distance r given in (3.2).

Given the above potential density, we can now calculate the force per unit p-brane volume for the probe as

$$f_p = -\frac{dV_p}{dr}$$

$$= -\frac{\alpha+\beta}{2} T_p \frac{\cosh^2\theta + \left(\frac{\tilde{H}}{H} \right)^{\alpha+\beta} \sinh^2\theta + 2 \left(\frac{\tilde{H}}{H} \right)^{(\alpha+\beta)/2} \sinh\theta\cosh\theta}{\left[\cosh^2\theta - \left(\frac{\tilde{H}}{H} \right)^{\alpha+\beta} \sinh^2\theta \right]^2}$$

$$\times \left(\frac{\tilde{H}}{H} \right)^{(\alpha+\beta)/2-1} \frac{2(7-p)\omega^{7-p}}{H^2 r^{8-p}}, \tag{3.11}$$

where we have used the explicit expressions for H and $\tilde{H}$ given in (3.2), and also the relations for α and β in (3.4). Given $\alpha + \beta \geq 0$ and $0 \leq \tilde{H}/H \leq 1$ for $r \geq \omega$, it can be checked easily that the above force is always attractive when $r > \omega$ as expected while at $r = \omega$ could be either zero or divergent depending solely on the sign of $(\alpha + \beta)/2 - 1$ since $\tilde{H}/H = 0$ at $r = \omega$ (i.e. at the location of the coincident Dp-$\bar{\text{D}}p$), and $1 \leq H \leq 2$ for $r \geq \omega$. Now to show this we write from (3.4),

$$\frac{\alpha+\beta}{2} - 1 = \sqrt{\frac{2(8-p)}{7-p} - \frac{(7-p)(p+1)}{16}\delta^2} - 1. \tag{3.12}$$

From what we have discussed below (3.7) regarding the validity of a probe when it is placed at $r = \omega$, we know that the δ parameter should be very close to its vanishing value at $T = 0$. With this and noting $0 \leq p \leq 6$ for well-behaved supergravity solutions, one can

---

[5]$\alpha + \beta \geq 0$ guarantees $F \geq 0$ and in that case the metric in (3.1) is well defined for $r \geq \omega$ as noticed in [18]. The parameter $\delta = 0$ at $T = 0$ previously pointed out in [10] prompted our discussion in [18] for the two disjoint decay channels of brane-antibrane systems, one in terms of open string tachyon condensation and the other in terms of the closed string tachyon condensation. In the former case the system ends up with a stable BPS configuration and while in the latter the system ends up with "bubble of nothing" through black brane. We will discuss these related issues in more detail elsewhere.

check, for example, taking $\delta = 0$ in the above that $(\alpha + \beta)/2 - 1 > 0$ for each allowed p. We thus notice from (3.11) that the force indeed vanishes. This, therefore, gives evidence for the static nature of the coincident Dp-$\bar{D}p$ system in the supergravity approximation as promised. We can actually do better for the parameter δ. Requiring $(\alpha + \beta)/2 - 1 > 0$, gives

$$|\delta| < \frac{4}{7-p}\sqrt{\frac{9-p}{p+1}}. \tag{3.13}$$

This bound allows the δ parameter to be deep in the bounded and the tachyon condensation region (i.e., far away from its vanishing value at $T = 0$ where the validity of the probe is guaranteed), therefore, providing even further evidence for the static nature of these solutions.

Given the property of a BPS p-brane supergravity configuration that a (BPS) probe p-brane will feel no-force at any transverse location when placed in this background and parallel to the large number of coincident source p-branes [24], enables us to obtain the stable BPS muti-p-brane configuration through a linear superposition of individual BPS p-brane configuration at different locations or placed coincidentally. With this, we expect that the force acting on a probe p-brane placed parallel to the source branes in the p-brane-anti p-brane background is due to the anti p-branes. When the probe is placed at the same location as the coincident branes in the p-brane-anti p-brane system, this probe brane acts the same as the brane in the brane-anti brane system in the tachyonic parameter region validating the probe approximation. So if the force acting on the probe vanishes, this may indicate that there is no-force acting between the coincident branes and anti branes in the brane-anti brane system in the supergravity approximation. Therefore, this provides an evidence to support the existence of static supergravity configuration describing the brane-anti brane system in the supergravity approximation. This is the rationale behind what we have shown above.

Now how to reconcile the result obtained here with the divergent force calculated in the previous section?[6] Before we address this, let us first point out the differences between these two scenarios. The supergravity description of the brane-antibrane system is obtained in the supergravity approximation where we consider only the massless modes in type II theories and their self-interaction (back-reaction) in the lowest order approximation. While the force calculation between a brane and an antibrane in the previous section counts all the modes but no back-reaction and the divergence is due to the tachyon mode when the brane separation is of the order of string scale. It is not difficult to check that if we count only the contribution of the massless modes to the force in the previous section (or for large brane separation where only the massless modes contribute), the result is always finite which is qualitatively consistent with the above probe calculation. Even in this case, we can only expect the two calculations to agree asymptotically in which the backreaction can be ignored and if the two systems can be prepared to be identical, i.e., a probe brane and a brane-antibrane system with a given separation. But the force acting on the probe in

---

[6]We again thank the referee for raising a pertinent question which has led to the discussion in this paragraph.

this section is evaluated at $r = \omega$, the location of the coincident branes-antibranes, where the two calculations have no way to agree, unlike the BPS case. So one should not directly compare the force calculated in this section to the one in the previous section.

The probe approach used in this section serves only the purpose of showing the static nature of brane-antibrane configuration in the supergravity approximation and should not be taken as a well-approximated calculation of brane-antibrane force in general. The rationale for doing this is explained in the Introduction and we will not repeat it here.

In summary, we have used a probe approach to provide a direct evidence to show that the force between the Dp and the $\bar{\text{D}}p$ in the coincident Dp-$\bar{\text{D}}p$ system in general vanishes. This, therefore, justifies the static nature of the general coincident Dp-$\bar{\text{D}}p$ configurations in supergravity and such static nature is due to the supergravity approximation.

Acknowledgments

References

[1] A. Sen, *Non-BPS states and branes in string theory*, `hep-th/9904207`.

[2] A. Sen, *Strong coupling dynamics of branes from M-theory*, *JHEP* **10** (1997) 002 [`hep-th/9708002`].

[3] O. Bergman, K. Hori and P. Yi, *Confinement on the brane*, *Nucl. Phys.* **B 580** (2000) 289 [`hep-th/0002223`].

[4] J.X. Lu and S. Roy, *Fundamental strings and NS5-branes from unstable D-branes in supergravity*, *Phys. Lett.* **B 637** (2006) 326 [`hep-th/0508045`].

[5] A. Sen, *Tachyon condensation on the brane antibrane system*, *JHEP* **08** (1998) 012 [`hep-th/9805170`].

[6] A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *JHEP* **03** (2000) 002 [`hep-th/9912249`];
N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [`hep-th/0002211`].

[7] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, *JHEP* **10** (2000) 034 [`hep-th/0009103`];
D. Kutasov, M. Mariño and G.W. Moore, *Remarks on tachyon condensation in superstring field theory*, `hep-th/0010108`; *Some exact results on tachyon condensation in string field theory*, *JHEP* **10** (2000) 045 [`hep-th/0009148`].

[8] M. Schnabl, *Analytic solution for tachyon condensation in open string field theory*, *Adv. Theor. Math. Phys.* **10** (2006) 433 [`hep-th/0511286`].

[9] P. Kraus and F. Larsen, *Boundary string field theory of the DD-bar system*, *Phys. Rev.* **D 63** (2001) 106004 [hep-th/0012198];
T. Takayanagi, S. Terashima and T. Uesugi, *Brane-antibrane action from boundary string field theory*, *JHEP* **03** (2001) 019 [hep-th/0012210].

[10] P. Brax, G. Mandal and Y. Oz, *Supergravity description of non-BPS branes*, *Phys. Rev.* **D 63** (2001) 064008 [hep-th/0005242].

[11] J.X. Lu and S. Roy, *Supergravity approach to tachyon condensation on the brane-antibrane system*, *Phys. Lett.* **B 599** (2004) 313 [hep-th/0403147].

[12] H. Bai, J.X. Lu and S. Roy, *Tachyon condensation on the intersecting brane-antibrane system*, *JHEP* **08** (2005) 068 [hep-th/0506115].

[13] J.X. Lu and S. Roy, *Non-susy p-branes delocalized in two directions, tachyon condensation and T-duality*, *JHEP* **06** (2005) 026 [hep-th/0503007].

[14] B. Zhou and C.-J. Zhu, *The complete black brane solutions in D-dimensional coupled gravity system*, hep-th/9905146.

[15] V.D. Ivashchuk and V.N. Melnikov, *Exact solutions in multidimensional gravity with antisymmetric forms*, *Class. and Quant. Grav.* **18** (2001) R87 [hep-th/0110274].

[16] Y.-G. Miao and N. Ohta, *Complete intersecting non-extreme p-branes*, *Phys. Lett.* **B 594** (2004) 218 [hep-th/0404082].

[17] J.X. Lu and S. Roy, *Static, non-SUSY p-branes in diverse dimensions*, *JHEP* **02** (2005) 001 [hep-th/0408242].

[18] H. Bai, J.X. LU and S. Roy, *Intersecting non-SUSY p-brane with chargeless 0-brane as black p-brane*, *JHEP* **01** (2007) 094 [hep-th/0610264].

[19] T. Banks and L. Susskind, *Brane - antibrane forces*, hep-th/9511194.

[20] J.X. Lu and S. Roy, *Non-SUSY p-branes, bubbles and tubular branes*, *Nucl. Phys.* **B 763** (2007) 170 [hep-th/0604048].

[21] S.P. de Alwis, *A note on brane tension and M-theory*, *Phys. Lett.* **B 388** (1996) 291 [hep-th/9607011].

[22] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[23] M.J. Duff and J.X. Lu, *Black and super p-branes in diverse dimensions*, *Nucl. Phys.* **B 416** (1994) 301 [hep-th/9306052].

[24] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rept.* **259** (1995) 213 [hep-th/9412184].